BUS 331 | INVESTMENTS

Derivatives & Risk Management

Options · Futures · Swaps

Ch 20: Options Intro	Ch 21: Option Valuation	Ch 22: Futures Markets	Ch 23: Futures, Swaps & Risk

What are they? | How are they valued? | How do we use them to manage risk?

CHAPTER 20

Options Markets: Introduction

- What is an Option?
- Put-Call Parity
- Strategies: Straddles & Spreads
- Payoffs & Profits
- Protective Put & Covered Call

What Is an Option?

CALL OPTION

Definition

The right to BUY an asset at the strike price (X) on or before expiration.

Profit Formula

Profit = max(S_T − X, 0) − Premium

Example

IBM $150 Call @ $5 premium. If S_T = $165 → Profit = $10

Unlimited upside, loss limited to premium

PUT OPTION

Definition

The right to SELL an asset at the strike price (X) on or before expiration.

Profit Formula

Profit = max(X − S_T, 0) − Premium

Example

IBM $150 Put @ $5 premium. If S_T = $135 → Profit = $10

Profit rises as price falls, loss limited to premium

In the Money | At the Money | Out of the Money | American vs. European

Option Strategies for Risk Management

Protective Put

Structure: Long stock + Long put
Use: Sets a floor on losses; portfolio insurance
Payoff: Upside unlimited; downside limited to (X − S□ + premium)

Covered Call

Structure: Long stock + Short call
Use: Generates income on existing holdings; caps upside
Payoff: Income = premium received; max gain = X − S□ + premium

Straddle

Structure: Long call + Long put (same X, same T)
Use: Profits from large moves in either direction
Payoff: Profit when |S_T − X| > combined premium

Collar

Structure: Long put + Short call (out-of-the-money)
Use: Brackets portfolio value between two bounds at low cost
Payoff: Loss floor AND gain cap; near zero-cost if strikes chosen carefully

Put-Call Parity

$$C + PV(X) = P + S_0$$

C
Call premium

PV(X)
PV of strike

P
Put premium

S_0
Current stock price

Portfolio A

- Long call (C)
- PV(X) invested at risk-free rate

At expiry: call converts to stock if S_T > X; otherwise receive X

Portfolio B

- Long stock (S_0)
- Long put (P)

At expiry: stock delivers S_T; put pays max(X − S_T, 0)

Identical payoffs ⇒ must have identical prices. Violation = arbitrage opportunity.

CHAPTER 21

Option Valuation

- Determinants of Option Value
- Binomial Option Pricing
- Black-Scholes Formula
- Delta & Hedge Ratios
- Portfolio Insurance
- Implied Volatility

Determinants of Option Value

Factor	Effect on Call	Effect on Put	Intuition
Stock Price (S)	Increases ↑	Decreases ↓	Higher S → deeper in the money for calls
Strike Price (X)	Decreases ↓	Increases ↑	Higher X → harder to profit for calls
Volatility (σ)	Increases ↑	Increases ↑	Both benefit — more chance of large moves
Time to Expiry (T)	Increases ↑	Increases ↑	More time = more uncertainty = more value
RiskFree Rate (r)	Increases ↑	Decreases ↓	Higher r → lower PV(X) → helps calls
Dividends (D)	Decreases ↓	Increases ↑	Dividends reduce future stock price

Key insight: Volatility (σ) is the only factor that increases BOTH call and put value—options are a bet on movement.

Black-Scholes Option Pricing Model

$$C = S_0 \cdot N(d_1) - X \cdot e^{-rT} \cdot N(d_2)$$

$$d_1 = [\ln(S_0/X) + (r + \sigma^2/2)T] / (\sigma\sqrt{T}) \qquad d_2 = d_1 - \sigma\sqrt{T}$$

S_0	X	r	T	σ
Stock Price	Strike Price	RiskFree Rate	Time (years)	Volatility
Current market price	*Exercise price*	*Continuous compounding*	*Days / 365*	*Annualized std dev of ln returns*

Worked Example

S_0=\$100 X=\$105 r=5% T=0.5yr σ=25% → d_1=0.122 d_2=−0.055 $N(d_1)$=0.549 $N(d_2)$=0.478

$C = 100(0.549) - 105 \cdot e^{-0.025}(0.478) = 54.9 - 48.6 =$ **\$6.30**

Delta, Hedge Ratios & Portfolio Insurance

Delta (Δ) — The Hedge Ratio

- $\Delta = N(d_1)$ from Black-Scholes
- = Slope of option price curve at S_0
- Δ = 0.55 → option moves \$0.55 per \$1 stock move
- Calls: $0 < \Delta < 1$ Puts: $-1 < \Delta < 0$
- At expiry: deep ITM call → $\Delta \approx 1$; OTM → $\Delta \approx 0$
- Delta changes as S changes → need dynamic rebalancing

Portfolio Insurance

Concept:	Use options or dynamic hedging to place a floor on portfolio value
Method 1:	Buy protective puts on index (direct; costly)
Method 2:	Synthetic put — dynamically adjust stock/T-bill mix based on delta
Rebalancing:	As S falls: sell stocks, buy T-bills (reduce exposure). As S rises: buy back.
Limitation:	Requires continuous rebalancing; tracking error; delta changes continuously

Implied Volatility: Backsolve Black-Scholes for σ given observed C. If IV > historical σ → option overpriced.

CHAPTER 22

Futures Markets

- The Futures Contract
- Trading Mechanics
- Speculation vs. Hedging
- Spot-Futures Parity
- Basis & Basis Risk
- Normal Backwardation vs. Contango

What Is a Futures Contract?

An agreement to buy or sell an asset at a specified future date at a price agreed upon today. Unlike options, futures obligate both parties.

	Futures	Forward
Standardized?	Yes	No (custom)
Exchange-traded?	Yes	OTC
Marked to Market?	Daily	At maturity
Clearinghouse?	Yes (guarantees)	No
Margin Required?	Yes (initial + maintenance)	None typically

Key Mechanics

- Long = agrees to BUY
- Short = agrees to SELL
- Daily mark-to-market → gains/losses settled each day
- Convergence: F → S as T → 0
- Most closed by reversal, not delivery

Profit on Long Futures = $F_T - F_0$ | Profit on Short Futures = $F_0 - F_T$

Futures Strategies: Speculation & Hedging

Speculation

- Bet on price direction using leverage
- Long futures → profit if price RISES
- Short futures → profit if price FALLS
- Control large contract value with small margin deposit
- Example: Long oil futures @ $80/bbl — if price rises to $90 → profit $10 × contract multiplier
- Risk: Unlimited loss potential (vs. options where loss = premium)

Hedging

- Offset existing price risk; exchange price risk for basis risk
- Short hedge: Sell futures to protect existing asset
- Long hedge: Buy futures to lock in purchase price
- Basis = Spot − Futures (narrows to zero at expiry)
- Example: Portfolio manager shorts S&P futures to hedge equity exposure
- Hedge ratio: $H^* = \rho \times (\sigma_S / \sigma_F)$ — minimize variance

Spot-Futures Parity (Cost-of-Carry)

$$F_0 = S_0 \times (1 + r\_f - d)^T$$

F_0 = futures price S_0 = spot price r_f = risk-free rate d = dividend yield T = time (years)

Futures Overpriced: $F_0 > S_0(1+r\_f-d)^T$

1. Sell (short) the futures contract
2. Buy the stock at spot price S_0
3. At delivery: deliver stock and receive F_0
4. Profit = $F_0 - S_0(1+r\_f-d)^T > 0$ (riskless)

Futures Underpriced: $F_0 < S_0(1+r\_f-d)^T$

1. Buy (long) the futures contract
2. Short-sell the stock at S_0; invest proceeds at r_f
3. At delivery: receive stock via futures at F_0
4. Profit = $S_0(1+r\_f-d)^T - F_0 > 0$ (riskless)

Arbitrage forces convergence — in equilibrium, no riskless profit is available.

CHAPTER 23

Futures, Swaps & Risk Management

- FX Futures & Interest Rate Parity
- Stock Index Futures
- Interest Rate Futures
- Interest Rate Swaps
- Credit Default Swaps
- Commodity Futures Pricing

Stock Index Futures: Hedging Equity Portfolios

Contracts to Short = (β_P / β_futures) × (Portfolio Value / Futures Value per Contract)

Worked Example: Hedge a $50M equity portfolio (β = 1.2) using S&P 500 futures (F = 4,500; multiplier = $250)

1. **Futures Contract Value**
   4,500× $250 = $1,125,000 per contract

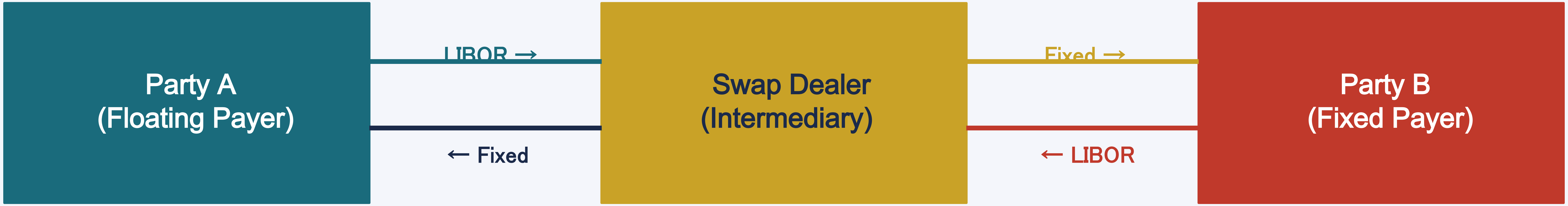
2. **Contracts Needed**
   N = (1.2 / 1.0) × ($50,000,000 / $1,125,000) = 53.3 ≈ 53 contracts

3. **Result**
   Short 53 S&P 500 futures contracts → portfolio beta effectively reduced to ≈ 0

Why futures instead of selling the stocks? Lower transaction costs, faster execution, no tax event, no portfolio disruption.

Interest Rate Swaps

An agreement to exchange a series of fixed-rate interest payments for floating-rate payments (or vice versa) on a notional principal.



Why Use?	Pricing	Risk
Transform floating-rate debt to fixed (or vice versa) without refinancing; reduces interest rate risk	Swap rate set so NPV = 0 at inception; a series of forward contracts on interest rates	Counterparty credit risk (mitigated by dealers); no exchange of principal—only net interest payments

CDS (Credit Default Swap): Like insurance on a bond—buyer pays periodic premium; seller compensates if credit event occurs.

Client Risk Management: Derivatives Toolkit

Equity Risk

Protective Put
Ch 20

Covered Call
Ch 20

Collar
Ch 20

Index Futures Short Hedge
Ch 23

Interest Rate Risk

Treasury Futures (short)
Ch 23

Interest Rate Swap (fixed)
Ch 23

Bond Options
Ch 20

Duration-Based Hedge Ratio
Ch 23

Currency / Commodity Risk

FX Forward / FX Futures
Ch 23

Covered Interest Arbitrage
Ch 23

Commodity Futures
Ch 23

Cross-Hedging
Ch 23

A well-structured derivatives program is not speculation—it is disciplined risk transfer.

Case Study: Constructing a Client Hedge Program

Client: Apex Capital – $100M endowment | 60% Equities / 30% Fixed Income / 10% Commodities | Fiscal year end in 6 months | Board requires no more than 8% drawdown

#	Risk / Hedge	Action	Reference
1	**Equity Drawdown** *Protective Put on S&P Index*	Buy 6-month S&P puts at 92% strike (8% OTM). Sets floor at −8%.	Ch 20 / Ch 21
2	**Rising Interest Rates** *Short Treasury Futures*	Short T-bond futures proportional to bond portfolio duration × value. Hedges duration exposure.	Ch 23
3	**FX Exposure (foreign equities)** *FX Forward Contracts*	Lock in current exchange rate for repatriation of foreign equity gains at fiscal year-end.	Ch 23
4	**Commodity Price Volatility** *Commodity Futures Hedge*	Short oil futures equal to current commodity allocation; hedge notional price exposure.	Ch 23

Key Takeaways: Chapters 20–23

Ch 20	Options give the right (not obligation) to buy/sell; strategies like protective puts and collars are direct tools for client risk management.
Ch 21	Black-Scholes prices options using 5 inputs; delta measures sensitivity; implied volatility reveals market sentiment; portfolio insurance is achievable synthetically.
Ch 22	Futures obligate both parties; spot-futures parity links cash and futures markets; hedgers exchange price risk for basis risk.
Ch 23	Index futures hedge equity beta; interest rate futures and swaps transform duration exposure; CDS transfers credit risk. Together these tools form a complete risk management toolkit.

Derivatives are not inherently risky—misuse is risky. Properly deployed, they are precision instruments for portfolio protecti